# Analytical model

In this section, we outline and clarify the analytical models in a step-by-step manner for two situations: (1) there is only a single node with batch arrival, and (2) there are three interconnected nodes with batch arrival in the network.

## Scenario 1

In this section, we focus on a scenario where packets coming with single or batch arrival can be divided into two priorities: high priority (HP) and low priority (LP), and both types of packets require one energy. Within each priority level, the packets are serviced in a first-come, first-served (FCFS) order. Once a packet enters the queue, it cannot be preempted, which means that an HP packet can always overtake an LP packet in the queue, but once an LP packet is in service, it cannot be interrupted. Additionally, there is a chance that a packet waiting in the queue may leave the system due to impatience. It is worth noting that when a packet is ready to be serviced, it first checks if there is enough energy in the energy queue. If there isn't enough energy, the packet may use a regular battery with a given probability. The model diagram, state balance equations, iterative algorithm, and performance metrics can be found below.

### Model diagram

Fig. 3 - 1 illustrates the components of the model used in scenario 1, which include a finite packet queue, a finite energy queue, a regular battery, and a single server. The size of the packet queue is denoted by , while the energy queue size is represented by . The regular battery has an infinite supply of energy, and each of HP and LP packets requires one energy unit. The arrivals of single HP batches (one packet per batch), single LP batches (one packet per batch), double HP batches (two packets per batch), double LP batches (two packets per batch), and energy units are governed by Poisson processes, with respective arrival rates , , , and . The impatient time for each HP and LP packet waiting in the queue is determined by an exponential distribution, with corresponding rates and . The service time for each of HP and LP packets in the server is exponentially distributed, with associated rates and , respectively. Additionally, when the amount of harvested energy available in the energy queue is insufficient to support an HP or LP packet, the regular battery will be used based on probabilities and , respectively.

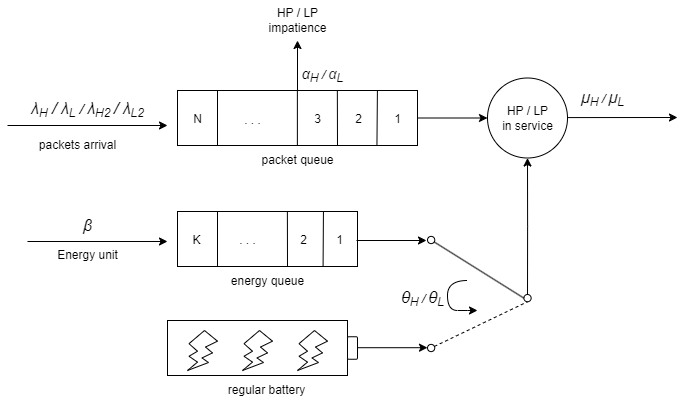


Fig. 3 - 1: The model diagram for scenario 1

### State balance equations

The system is modeled as a Markov chain with four dimensions: , where represents the number of high-priority (HP) packets in the system, represents the number of low-priority (LP) packets in the system, represents the number of energy units in the energy queue, and represents the server status and the energy source used. The value of can take on five different values: (1) "" indicates that the server is idle; (2) "" indicates that an LP packet has entered the server and consumed one energy unit from the energy queue; (3) "" indicates that an HP packet has entered the server and consumed one energy unit from the energy queue; (4) "" indicates that an LP packet has entered the server and consumed one energy unit from the regular battery; (5) "" indicates that an HP packet has entered the server and consumed one energy unit from the regular battery. The steady state probability of the system is represented by , and the state space is defined as follows:

(3-1)

As a result, we can calculate the total count of possible states  
.

Based on the model description, there are 190 possible cases for the total system states. The balance equations for each of these cases are presented below.

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1. For,,,and,

Since there are many equations presented above, discussing each one separately would be challenging. Therefore, we focus on a relatively complicated case, specifically case 159, to provide an illustrative example. This state occurs when there are more than or equal to three but less than or equal to N-3 HP packets and N-i LP packets in the system, and there is only one seat left in the packet queue, while the energy queue is empty. The HP packet being served in the server is using the regular battery. The corresponding detailed state transition diagram can be found in Fig. 3 - 2.

1. For,,,and,

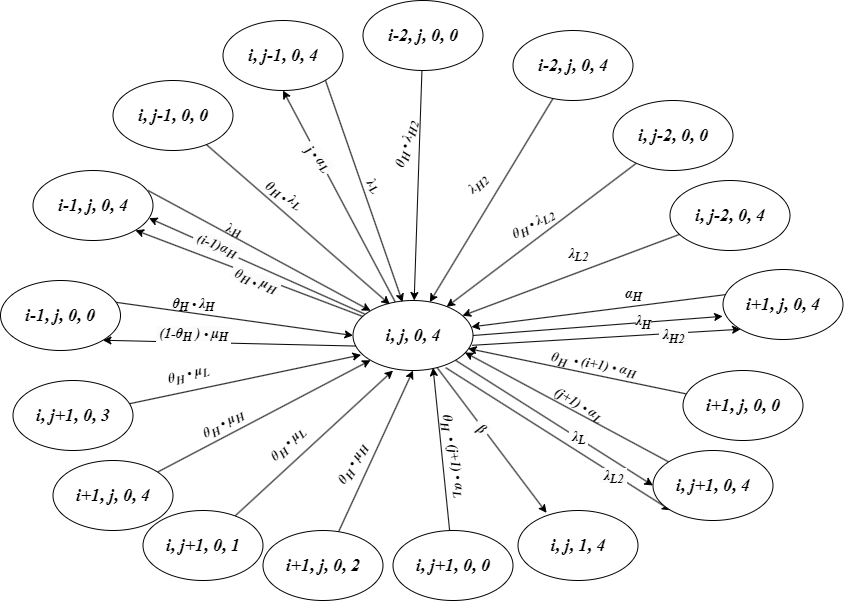


Fig. 3 - 2: The state transition diagram for ,,,and *.*

### Iterative algorithm

Using the iterative algorithm provided below, we perform calculations on the state balance equations until they converge, allowing us to determine the steady-state distribution of the system.

#### **Iterative algorithm:**

Step 1: Select a group of initial values for , , where is the total number of feasible states.

Step 2: Substitute into *Case 1* to *Case 190* to find , .

Step 3: Normalize , .

Step 4: If , stop the iterative algorithm, where is the stopping criterion. Otherwise, set , and return to Step 2.

In the analytical experiments, we set . It takes about 200 to 7000 iterations for the algorithm to converge.

### Performance measures

We obtain different performance measures of interest from the steady-state probability in order to evaluate the system's measures of effectiveness. These measures are presented below.

#### , the expected number of all packets in the system, is provided below.

(3-2)

(3-3)

(3-4)

#### , the expected number of all packets in the queue, is provided below.

(3-5)

(3-6)

(3-7)

#### , the throughput of all packets, is provided below.

(3-8)

(3-9)

(3-10)

#### , the blocking probability of each arrived packet, without considering its priority, is provided below.

(3-11)

#### , the energy loss probability, is provided below.

(3-12)

#### , the mean waiting time of all packets in the system, , which refers to all packets that have exited the system, either after receiving service or due to impatience, is provided below.

(3-13)

(3-14)

(3-15)

#### , the impatient loss probability of arrived packets, is provided below.

(3-16)

(3-17)

(3-18)

#### , the impatient loss probability of admitted packets, is provided below.

(3-19)

(3-20)

(3-21)

#### , the total loss probability of arrived packets, is provided below.

(3-22)

(3-23)

(3-24)

#### , the regular energy consumption ratio of all packets, is provided below.

(3-25)

(3-26)

(3-27)